Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions:

There are EIGHT questions divided in TWO SECTIONS and printed both in HINDI and in ENGLISH.

Candidate has to attempt FIVE questions in all.

Questions no. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each section.

The number of marks carried by a question / part is indicated against it.

Answers must be written in the medium authorized in the Admission Certificate which must be stated clearly on the cover of this Question-cum-Answer (QCA) Booklet in the space provided. No marks will be given for answers written in a medium other than the authorized one.

Assume suitable data, if considered necessary, and indicate the same clearly.

Unless and otherwise indicated, symbols and notations carry their usual standard meanings.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.
SECTION A

Q1. (a) A rocket starts vertically upward with speed $v_0$. Then define its speed $v$ at a height $h$ in terms of $v_0$, $h$, $R$ (radius of Earth) and $g$ (acceleration due to gravity on Earth’s surface). Also calculate the maximum height attained by a rocket fired with a speed of 90% of the escape velocity.

(b) Determine the location of the centre of mass of a uniform solid hemisphere of radius $R$ and mass $M$ from the centre of its base.

(c) A rubber cord 1 mm in diameter and 1 m long is fixed at one end and a weight of 1 kg is attached to the other end. If the Young’s modulus of rubber is $0.05 \times 10^{11}$ dynes cm$^{-2}$, then find the period of the vertical oscillations of the weight.

(d) What are Newton’s rings? How are they formed by two curved surfaces?

(e) The equation for displacement ($X$) of a point on a damped oscillator is given by

$$X = 5 e^{-0.25t} \sin \left( \frac{\pi}{2} \right) t$$

Find the velocity of oscillating point at $t = \frac{T}{4}$ and $T$, where $T$ is the time period of the oscillator. What is the direction of velocity in each case?
Q2. (a) Obtain expressions for the moment of inertia of a solid cone about its (i) vertical axis and (ii) axis passing through the vertex and parallel to its base.

(b) A shaft of diameter 8 cm and length 5 m is transmitting power of 8 kW at 300 revolutions per minute. If the coefficient of rigidity of the material of the shaft be $8 \times 10^{11}$ dynes/cm², then calculate the relative shift between the ends of the shaft.

(c) What do you understand by length contraction? Calculate the percentage length contraction of a rod moving with a velocity 0.8 c in a direction inclined at 60° with respect to its own length.

Q3. (a) Discuss the conditions for interference. Describe Young's double-slit experiment and derive an expression for the estimation of fringe width. Discuss its dependency on various parameters.

Green light of wavelength 5100 Å from a narrow slit is incident on a double-slit. If the overall separation of 10 fringes on a screen 200 cm away is 2 cm, find the slit separation.
(b) A zone plate is used in optical microscopy. Explain why. It is known that a zone plate has multiple foci. Differentiate a zone plate from a convex lens.

What is a zone plate? Give its theoretical description. Show that a zone plate has multiple foci. Differentiate a zone plate from a convex lens.

Calculate the radius of the first half period zone in a zone plate behaving like a convex lens of focal length 60 cm for light of wavelength 6000 Å.

(c) Derive the expression for the wavelength of light in a medium of refractive index n, using the relation between velocity and frequency.

Briefly discuss the postulates of Einstein to explain stimulated emission. Derive an expression for Einstein’s A and B coefficients and show that the ratio of coefficients of spontaneous versus stimulated emission is proportional to the third power of frequency of radiation. Why is it difficult to achieve laser action in higher frequency ranges such as X-rays?

Can there be a temperature at which the rates of spontaneous and stimulated emission are equal? Illustrate with wavelength \(\lambda = 5000 \text{ Å}\).

Q4. (a) Derive the relativistic expression for kinetic energy by considering mass variation with velocity. Hence, establish the relation between momentum (p) and energy (E) for a relativistic particle; \(\frac{dE}{dp} = v\).
(b) चरम मार्ग (पथ) के फर्मा के सिद्धांत का उद्देश्य कीजिए और उसकी व्याख्या कीजिए। फर्मा के सिद्धांत के संदर्भ में प्रकाश के सततेसीय संचरण एवं प्रकाश की किरणों की उत्क्रमणीयता की चर्चा कीजिए। फर्मा के सिद्धांत का उपयोग करते हुए पतले लेंस (thin lens) के सूत्र की उत्पत्ति कीजिए।

State and explain Fermat’s principle of extremum path. Discuss the cases of rectilinear propagation of light and reversibility of light rays in context of Fermat’s principle. Using Fermat’s principle, deduce the thin lens formula.

(c) कैल्साइट क्रिस्टल में द्वि-अपवर्तन की परिघटना को समझाइए। द्वि-अपवर्ती क्रिस्टल को कुथालक पदार्थ मान कर विद्युत-चुम्बकीय सिद्धांत का उपयोग करते हुए द्वि-अपवर्तन की व्याख्या कीजिए।

एक ऐसी द्वि-अपवर्ती प्लेट की मोटाई की गणना कीजिए जो कि साधारण एवं असाधारण तरंगों के बीच $\frac{\lambda}{4}$ का पथांतर उत्पन्न करती है।

दिया गया है:

$$\lambda = 5890 \, \text{Å}, \quad \mu_0 = 1.53, \quad \mu_e = 1.54$$

Explain the phenomenon of double refraction in calcite crystal. Considering birefringent crystal as non-conducting material, explain double refraction using electromagnetic theory.

Calculate the thickness of a double refracting plate which produces a path difference of $\frac{\lambda}{4}$ between extraordinary and ordinary waves.

Given:

$$\lambda = 5890 \, \text{Å}, \quad \mu_0 = 1.53, \quad \mu_e = 1.54$$
Q5. (a) A vertically oriented electric dipole having dipole moment \( \vec{p} \) is kept at height \( h \) above an infinitely large horizontal conducting plate, which is grounded as shown in the diagram. Calculate the force between the electric dipole and the conducting plate by using method of images.

(b) For the electric field given by \( E = E_0 e^{iot} \), show that the conduction current is in phase with the electric field, while the displacement current leads the electric field by \( \frac{\pi}{2} \) radians. Also, show that the displacement current in a good conductor is negligible compared to the conduction current at any frequency lower than the optical frequencies (\( f < 10^{15} \) Hz).
Based on the hysteresis loops for soft iron and steel as shown in the diagram, which material would you prefer to utilise for making transformer cores and why?
(d) एक विस्तृत अन्तःक्रिया वाले निकाय के लिए ऊष्मागतिकी के प्रथम नियम का उपयोग कीजिए। स्थिर आयतन पर 10 g हवा का तापक्रम 2°C बढ़ाया जाता है। उसकी आंतरिक ऊर्जा में वृद्धि की गणना कीजिए।

दिया गया है: \( C_v = 0.172 \text{ cal g}^{-1} \text{ °C}^{-1} \)

State the first law of thermodynamics for a diffusively interacting system. The temperature of 10 g of air is raised by 2°C at constant volume. Calculate the increase in its internal energy.

Given: \( C_v = 0.172 \text{ cal g}^{-1} \text{ °C}^{-1} \)

(e) (i) एक \( \nu \) आवृत्ति के क्वांटम आवृत्ति (हामिल्कन) दोलक के ऊर्जा स्तर इस प्रकार दिए गए हैं:

\[ E_n = \left( n + \frac{1}{2} \right) \nu, \text{ जहाँ } n = 0, 1, 2, ... \]

इसका संविदर्ण फलन परिकलित कीजिए।

The energy level of a quantum harmonic oscillator with frequency \( \nu \) is given by

\[ E_n = \left( n + \frac{1}{2} \right) \nu, \text{ where } n = 0, 1, 2, ... \]

Calculate its partition function.

(ii) एक द्विस्तरीय निकाय का संविदर्ण फलन परिकलित कीजिए।

Calculate the partition function of a two level system.

Q6. (a) स्थिर-वैधुत क्षेत्र के लिए डाइवर्जेंस (अपस्तरण) और कूल्ड के व्यंजक लिखिए। इससे प्यासों और लागतास समीकरणों को प्राप्त कीजिए।

विश्लेषों \( r_1 \) और \( r_2 \) \((r_1 < r_2)\) की दो संकेतित सूचकात गोलीय कोशों को क्रमशः \( V_1 \) और \( V_2 \) विभव पर आवृत्ति किया जाता है। उन दोनों कोशों के बीच अंतराल में विधुत-विभव और अतः विधुत-क्षेत्र की गणना कीजिए। आंतरिक कोश पर आवृत्ति की मात्रा को भी ज्ञात कीजिए।

Write expressions for divergence and curl of an electrostatic field. From these, obtain Poisson and Laplace equations.

Two concentric conducting spherical shells having radii \( r_1 \) and \( r_2 \) \((r_1 < r_2)\) are charged to potentials \( V_1 \) and \( V_2 \), respectively. What are the electric potential and hence electric field in the space between the shells? Also find the charge on the inner shell.
(b) The oscillations of electric and magnetic fields in an ideal LC circuit.

Describe the oscillations of electric and magnetic fields in an ideal LC circuit.

The applied voltage phasor in a circuit is \((4 + 3i)\) volt and resulting current phasor is \((3 + 4i)\) ampere. Draw the phasor diagram. Determine the impedance of the circuit and indicate whether it is inductive or capacitive in nature. Also find the power dissipation in the circuit.

(c) For free space show that electromagnetic (EM) wave is transverse in nature. Show that for free space, the total outward flux of EM energy through surface \(S\) bounding volume \(V\) is equal to the rate of loss of EM energy from the volume \(V\).

A laser beam of 2 mm diameter has average power of 20 GW. Calculate the peak values of electric and magnetic fields in the laser beam.

Q7. (a) Discuss the principle of adiabatic demagnetization process to achieve low temperatures.

Determine the fall in temperature produced by adiabatic demagnetization of a paramagnetic material at initial temperature of 3 K when the magnetic field is switched off from 10,000 oersted to zero. Given: heat capacity at constant magnetic field = 0.2 J g\(^{-1}\) K\(^{-1}\) and Curie constant per gram mole per cm\(^3\) = 0.042 erg K\(^{-1}\) g\(^{-1}\) Oe\(^{-2}\).
(b) A molecule of water is a diatom. The water molecule (H₂O) can be viewed as a two-dimensional object with two hydrogen atoms bonded to an oxygen atom. The molecule is electrically neutral, with an overall dipole moment due to the unequal distribution of charge. The hydrogen bonds between water molecules are strong due to the polar nature of the water molecule.

Starting from Maxwell-Boltzmann distribution for a free particle in 3-dimensional space, obtain the expression for root mean square (rms) speed of a particle. Calculate the rms speed of nitrogen (N₂) molecule at room temperature (27°C).

(c) Nonlinear sorption isotherms are used in industry to represent the adsorption of gases on solid surfaces. The Langmuir isotherm is a mathematical model that describes the adsorption of gases on a solid surface. It is given by the equation:

\[ Q = \frac{Q_m b P}{1 + b P} \]

where \( Q \) is the amount of adsorbed gas, \( Q_m \) is the maximum amount of gas that can be adsorbed, \( b \) is the affinity constant, and \( P \) is the pressure of the gas.

Obtain the Clausius-Clapeyron equation. Using this equation, show that for the phase boundary of the liquid and vapour phases, the relation can be written as:

\[ p = p_0 e^{-\frac{L}{R T}} \]

Here it has been assumed that the latent heat \( L \) is independent of temperature, that vapour is treated as an ideal gas, and that \( V_{\text{vapour}} = V >> V_{\text{liquid}} \) and that \( p \to p_0 \) as \( T \to \infty \).

Q8. (a) Classical physics refers to the fundamental laws of nature that describe the behavior of matter and energy, particularly in the context of macroscopic objects. Classical physics is based on the principles of Newtonian mechanics, electromagnetism, thermodynamics, and quantum theory.

(i) Causal set theory is a framework that attempts to describe the universe as a collection of discrete, indecomposable elements called points, or vertices. These points are connected by directed links, which represent the direction of time.

(ii) Quantum mechanics is a fundamental theory in physics that describes the physical properties of nature at the scale of atoms and subatomic particles. It is a probabilistic theory, meaning that the outcomes of experiments are not certain but are described by probabilities.

\[ URC-U-PHY \]
A historic failure of Classical Physics is its inability to describe the electromagnetic radiation emitted from a black body. Consider a simple model for an ideal black body consisting of a cubic cavity of side L with a small hole on one side.

(i) Assuming the classical equipartition of energy, derive an expression for the average energy per unit volume and unit frequency range. In what way does this result deviate from actual observation? What is this law called?

(ii) Repeat the calculations now using quantum idea to obtain an expression that properly accounts for the observed spectral distribution. Find the temperature dependence of the total power emitted from the hole.

(b) निर्वैत में मैक्सवेल समीकरणों को अवकल एवं समाकल दोनों रूपों में लिखिए। तरंग समीकरणों को प्रावध कीजिए एवं दिखाइए कि विद्यु-चुमुकीय तरंग निर्वैत में प्रकाश की गति से गतिमान हो सकती है। क्या मैक्सवेल समीकरणों के समाकल रूप से तरंग समीकरणों को प्रावध किया जा सकता है?

Write Maxwell’s equations in free space in both differential and integral forms. Obtain wave equations and show that electromagnetic waves can travel in free space with a speed of light. Can one get the wave equations from the integral form of the Maxwell’s equations?

(c) एक 10Ω का प्रतिघंट, एक 1-0 µF के संधारित्र और एक विद्यु-चुमुक बल 12-0 V की बैटरी से श्रेणीब्रेकम में सम्बंद है। समय t = 0 पर विद्यु बन्द करने से पहले संधारित्र अनावेशित है।

निम्नलिखित का आकलन कीजिए:

(i) समय नियतांक।
(ii) समय t = 46 सेकंड पर प्लेटों पर अंतिम आवेश का कितना भाग है?
(iii) समय t = 46 सेकंड पर प्रारम्भिक धारा का कितना भाग रहता है?

मान लीजिए कि बैटरी का आंतरिक प्रतिघंट शून्य है तथा सभी जुड़े हुए तारों का प्रतिघंट नगण्य है।
A 10 Ω resistor is connected in series with a capacitor of 1.0 μF and a battery with emf 12.0 V. Before the switch is closed at time \( t = 0 \), the capacitor is uncharged.

Calculate the following:

(i) The time constant.

(ii) What fraction of the final charge is on the plates at the time \( t = 46 \) seconds?

(iii) What fraction of the initial current remains at the time \( t = 46 \) seconds?

Consider that the internal resistance of the battery is zero and neglect the resistance of all the connecting wires.