IES/ISS EXAM, 2014

A-HDR/HRR-N-TUB

STATISTICS

Paper II

Time Allowed : Three Hours | *Maximum Marks : 200*

INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are EIGHT questions divided under TWO sections.

Candidate has to attempt SIX questions in all.

Question No. 1 and 5 are compulsory and out of the remaining, FOUR are to be attempted choosing at least TWO from each Section.

The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Candidates should attempt questions/parts as per the instructions given in the Section.

All parts and sub-parts of a question are to be attempted together in the answer book.

Attempts of questions shall be counted in chronological order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the answer book must be clearly struck off.

Answers must be written in ENGLISH only.

Section – A

1. Answer all of the following : 5×8=40

(a) $x_1, x_2, \ldots x_n$ be a random sample from $U(0, \theta)$ obtain the moment estimator of θ. Also find its variance.

(b) Obtain a g-inverse (A^-) of A given below and verify that $AA^-A = A$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}.$$

(c) Define completeness. Verify whether Bin $(1, p)$ is complete.

(d) Find the sum of squares due to

$$\left(\sum_{i=1}^{p} \sum_{j=1}^{q} a_i\, b_j\, y_{ij} \right) \Big/ \left(\sum_{i=1}^{p} a_i^2 \right) \left(\sum_{j=1}^{q} b_j^2 \right)$$

under suitable assumptions on y_{ij}'s where a_i and b_j's are constants. State its use in Analysis of variance.

(e) Let X_1, X_2, X_3 and X_4 be four random variables such that

$$E(X_1) = \theta_1 - \theta_3,\ E(X_2) = \theta_1 + \theta_2 - \theta_3,$$

$$E(X_3) = \theta_1 - \theta_3,\ E(X_4) = \theta_1 - \theta_2 - \theta_3$$

where $\theta_1, \theta_2, \theta_3$ are unknown parameters. Assume var $(X_i) = \sigma^2$, $i = 1, 2, 3, 4$. Check if θ_2 is estimable. If so obtain its BLUE.

(f) $x_1, x_2, \ldots x_n$ is a random sample from the following distribution

$$f(x, \alpha) = e^{-(x-\alpha)}, \qquad x \geq \alpha;$$
$$= 0 \qquad \text{otherwise.}$$

Find MLE of α.

(g) Let $x_1, x_2, \ldots\ldots x_n$ be a random sample from $U[0, \theta]$. Let $T = \left(\prod_{i=1}^{n} x_i \right)^{1/n}$. Is T an unbiased estimator of θ ? If not suggest an unbiased estimator of θ which is a function of T.

(h) Let X be exponentially distributed with parameter θ. Obtain MLE of θ based on a sample of size n, from the above distribution.

2. Answer *all* of the following : 10×3=30

(a) Define estimability of a linear parametric function in a Gauss Markoff model. State and prove a necessary and sufficient condition for estimability.

(b) For the Pareto distribution with pdf

$$f(x, \lambda) = \frac{\lambda}{x^{\lambda+1}} \qquad x \geqslant 1, \lambda > 0$$

Show that method of moments fails if $0 < \lambda < 1$. State the method of moments estimator when $\lambda > 1$. Is it consistent ? Justify your answer.

(c) Consider the model with normal assumption on error variables.

$$\begin{aligned} E(y_1) &= \theta_1 - \theta_2 + \varepsilon_1 \\ E(y_2) &= \theta_2 - \theta_3 + \varepsilon_2 \\ &\vdots \\ E(y_n) &= \theta_n - \theta_1 + \varepsilon_n \end{aligned}$$

Find the error function(s) and the BLUE of $\theta_1 - \theta_2$.

3. Answer *all* of the following : 10×3=30

(a) A manufacturer of television sets is interested in the effect of tube conductivity of five different types of coating for colour picture tubes. Sample means are

$\bar{y}_{1.} = 49, \bar{y}_{2.} = 51, \bar{y}_{3.} = 53, \bar{y}_{4.} = 57, \bar{y}_{5.} = 58.$

Error Mean sum of squares = 54·0

(i) Is there a difference in coating for colour picture tubes ? F value = 3·09.

(ii) Determine which pairs differ significantly using Bonferroni t-interval. Comment on the same.

$$\left[t_{15}\left(\frac{0{\cdot}05}{20}\right) = 3{\cdot}286\right].$$

(b) $X_1, X_2, \ldots X_n$ are i.i.d. random variables from $N(\theta, 1)$ where θ is an integer. Obtain MLE of θ.

(c) Suppose

$$E\left(Y_{ij}\right) = \alpha_i + \beta_i X_{ij}, \quad 1 \leq j \leq n_i,\ 1 \leq i \leq K$$

where Y_{ij} are independent homoscedastic normal variables, X_{ij}'s are non stochastic and α_i and β_i are unknown parameters.

(i) Find a suitable test of $H_{01} : \beta_1 = \beta_2 = .. = \beta_K$

(ii) Assuming $\beta_1 = \beta_2 .. = \beta_K$, derive a test for $H_{02} : \alpha_1 = \alpha_2 .. = \alpha_K$.

4. Answer *all* of the following : 10×3=30

(a) Obtain unbiased estimators of the variance components in two way classification model with interaction, with r observations per cell when there are S_1 levels of factor A and S_2 levels of factor B.

(b) $x_1, x_2, \ldots x_n$ be a random sample from a population having pmf

$$P_N(x) = \begin{cases} \frac{1}{N} & \text{if } x = 1, 2 \ldots N \\ 0 & \text{otherwise} \end{cases}$$

Derive UMVUE of N.

(c) Show that $Z = \frac{1}{6}(X_1 + 2X_2 + 3X_3)$ is not a sufficient estimator of the Bernaulli parameter θ.

Section – B

5. Answer *all* of the following : 5×8=40

(a) Let X be r.v. with pmf under H_0 and H_1 given below. Find M.P. test with α = ·03

x	1	2	3	4	5	6
$f_0(x)$	0·01	0·01	0·01	0·01	0·01	0·95
$f_1(x)$	0·05	0·04	0·03	0·02	0·01	0·85

(b) A single observation of a r.v. having a geometric distribution with pmf

$$f(x, \theta) = \theta (1-\theta)^{x-1}, \quad x = 1, 2, 3, \ldots..$$
$$= 0 \quad \text{otherwise.}$$

The null hypothesis $H_0 : \theta = 0{\cdot}5$ against the alternative hypothesis $H_1 : \theta = 0{\cdot}6$ is rejected if the observed value of the r.v. is greater than or equal to 5. Find probabilities of type I error and type II error.

(c) The pdf of 5 variate normal distribution is given by :

$$\frac{1}{(2\pi)^{5/2} - 30} \exp\left\{-\frac{1}{2}\left[(x_1-5)^2 + \frac{(x_2-2)^2}{9} + \frac{x_3^2}{5} + \frac{(x_4+1)^2}{4} + \frac{x_5^2}{5}\right]\right\}$$

Obtain mean vector and variance covariance matrix of the distribution.

(d) Let $X \sim N_3(0, \Sigma)$, $\Sigma = \begin{bmatrix} 1 & \rho & 0 \\ \rho & 1 & \rho \\ 0 & \rho & 1 \end{bmatrix}$

Find ρ such that $X_1 + X_2 + X_3$ and $X_1 - X_2 - X_3$ are independent.

(e) Find 3-sigma control limits for a

(i) C chart with process average equal to 4 nonconformities.

(ii) U chart with process average $c = 4$ and $n = 4$.

(f) Justify the statement :

P chart is equivalent to Chi-square test of homogenity.

(g) A sample of size n from normal distribution $N(\theta, \sigma^2)$ with $\sigma^2 = 4$ was observed. 95% confidence interval for θ was computed from the above sample. Find the value of n if the confidence interval is (9·02, 10·98).

(h) Bring out the difference between a randomized test and a nonrandomized test. Explain how the decision based on a randomized test can be taken in the discrete set up.

6. Answer ***all*** of the following : 10×3=30

(a) Consider the Hypothesis $H_0 : p = \frac{1}{2}$ against $H_1 : p = 1$ for a binomial X for which $n = 2$. List all possible critical regions for which $\alpha \leq \frac{1}{2}$. Which of these regions minimizes $\alpha + \beta$.

(b) Consider an i.i.d. sample of size $n = 5$ from bivariate normal distribution.

$$X \sim N_2\left(\mu, \begin{bmatrix} 3 & a \\ a & 1 \end{bmatrix}\right) \cdot \quad \bar{X}' = [0, 0 \cdot 5]$$

For what values of a would the hypothesis $H_0 : \mu = (0, 0)'$ be rejected in favour of $H_1 : \mu \neq (0, 0)'$ at 5% level of significance. (Chi-square table value = 5·99).

(c) Obtain expressions for OC and ASN function under (*i*) Single sampling plan (*ii*) Double sampling plan.

7. Answer ***all*** of the following : 10×3=30

(a) Let $X \sim N_3(\mu, \Sigma) \cdot \quad \Sigma = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 6 & 0 \\ -1 & 0 & 4 \end{bmatrix}$

Set
$Y_1 = X_1 + X_3$, $Y_2 = 2X_1 - X_2$ and $Y_3 = 2X_3 - X_2$. Find the conditional distribution of Y_3 given $Y_1 = 0$, $Y_2 = 1$.

(b) $x_1, x_2, \ldots x_n$ is a random sample from $N(\theta, \sigma^2)$ (σ^2 not specified). Derive likelihood ratio test of testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

(c) A $\overline{X}$ chart is used to control the mean of a normally distributed quality characteristic. It is known that $\sigma = 6$ and $n = 4$. The center line is 200. If the process mean shifts to 188, find the probability that this shift is detected on the first subsequent sample.

8. Answer ***all*** of the following : 10×3=30

(a) Let X be a r.v. following exponential distribution

$$f(x, \theta) = \theta e^{-\theta x} \qquad x > 0$$
$$= 0 \qquad \text{otherwise}$$

Obtain SPRT of strength α, β for testing $H_0 : \theta = 8$ against $H_1 : \theta > 8$

(b) $$(X_1, X_2, X_3) \sim N_3\left(\underline{0}, \begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ & 1 & \rho_{23} \\ & & 1 \end{bmatrix}\right)$$

Show that $1 + 2\rho_{12}\rho_{13}\rho_{23} \geq \rho_{12}^2 + \rho_{13}^2 + \rho_{23}^2$

(c) A proposed triple sampling plan is as follows :

Take a first sample of 2 articles. If both good accept, if both bad reject. If 1 good and 1 bad reject, take a second sample of 2. If both good accept. If both bad reject. If 1 good and 1 bad take a third sample of 2. If both articles in the third sample are good, accept; otherwise reject. Find the OC of this plan assuming that a large lot of $p\%$ defective is submitted for inspection.
